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The Proof of the Generalized Riemann hypothesis , Riemann conjecture, Polignac conjecture, twin prime conjecture ,and Goldbach conjecture

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Abstract: In order to strictly prove the hypothesis and conjectures in Riemann's 1859 paper on the Number of Prime Numbers Not greater than x from a pure mathematical point of view, and in order to strictly prove the Generalized hypothesis and the Generalized conjectures, this paper uses Euler's formula to study the relationship between symmetric and conjugated zeros of Riemann's ζ(s) function and Riemann's ξ(s) function, and proves that Riemann's hypothesis and Riemann's conjecture are completely correct.

Key words: *Euler's formula, Riemann ζ(s) function, Riemann function ξ(t), Riemann hypothesis, Riemann conjecture, symmetric zeros, conjugate zeros, uniqueness.*

I. Introduction

Riemann hypothesis and Riemann conjecture are an important and famous mathematical problem left by Riemann in his paper "On the Number of prime Numbers not greater than x" [1], which is of great significance for the study of prime number distribution and known as the biggest unsolved mystery in mathematics. After years of hard work, I have solved this problem and strictly prove the Generalized hypothesis and the Generalized conjectures, The research shows that the Riemann hypothesis and the Riemann conjecture and the Generalized Riemann hypothesis and the Generalized Riemann conjecture are all completely valid and the Polignac conjecture,twin prime conjecture and Goldbach conjecture are completely true.

II. ConclusionReasoning

$\sum_n n^{-s}=\prod_p (1-p^{-s})^{-1}$($n\in Z_+, p\in Z_+, s\in C$, n goes through all the natural numbers, p goes through all the prime numbers),this formula was proposed and proved by the Swiss mathematician Leonhard Euler in 1737 in a paper entitled "Some Observations on Infinite Series", Euler's product formula connects a summation expression for natural numbers with a continuative product expression for prime numbers, and contains important information about the distribution of prime numbers. This information was finally deciphered by Riemann after a long gap of 122 years, which led to Riemann's famous paper "On the Number of primes less than a Given Value [1]. In honor of Riemann, the left end of the Euler product formula was named after Riemann, and the notation ζ(s) used by Riemann was adopted as the Riemann zeta function .

Because e=2.718281828459045... ,e is a natural constant, I use * for Multiplication, ^ for multiplication,then based on euler's e^{ix}=cosx+isin(x) (x∈R),

get (e^(3i))^(2)=(cos(3)+isin(3))^(2)=cos(2*3)+isin(2*3)=cos(6)+isin(6) ,

because e^(6i)=cos(6)+isin(6),

so

(e^(3i))^(2)= e^(6i) ,

In general,

(e^(bi))^(c)= e^(b*ci)(b∈R， c∈R) is established.

When x>0(x∈R),suppose e^j=x (e=2.718281828459045… , x∈R and x>0 , j∈R) , then

j=ln(x),based on euler's e^{ix}= cos(x)+isin(x) (x∈R),will get

$e^{ji} = e^{\ln(x)i}$=cos(lnx)+isin(lnx)(x∈R and x>0).

suppose y∈R and $y \neq 0$, now let's figure out expression for x^{yi}(x∈R, and x>0, y∈R and $y \neq 0$) is $x^{yi}=(e^j)^{yi}=(e^{ji})^y=(\cos(\ln x) + i\sin(\ln x))^y$.

Suppose s is any complex number, and s=ρ+yi (ρ∈R,y∈R and $y \neq 0$,s∈C) ,then let's find the expression of x^s(x∈R and x>0, s∈C) ,

You put s=ρ+yi (ρ∈R,y∈R and $y \neq 0$,s∈C) and $x^{yi}=(e^j)^{yi}=(e^{ji})^y=(\cos(\ln x) + i\sin(\ln x))^y$ into x^s and you will get

$x^s=x^{(\rho+yi)}=x^\rho x^{yi}=x^\rho(\cos(\ln x) + i\sin(\ln x))^y=x^\rho(\cos(y\ln x) + i\sin(y\ln x))$,

You put s=ρ-yi (ρ∈R,y∈R and $y \neq 0$,s∈C) and $x^{yi}=(e^j)^{yi}=(e^{ji})^y=(\cos(\ln x) + i\sin(\ln x))^y$ into x^s and you will get

$x^{\bar{s}} = x^{(\rho-yi)} = x^\rho x^{-yi} = x^\rho(x^{yi})^{-1} = x^\rho(\cos(\ln x) + i\sin(\ln x))^{-y}=x^\rho(\cos(-y\ln x) + i\sin(-y\ln x))=x^\rho(\cos(y\ln x) - i\sin(y\ln x))$.

For any complex number s,when $Rs(s) > 0 \wedge (s \neq 1)$,then according to Dirichlet function

$\eta(s)=\sum_1^\infty \frac{(-1)^{n-1}}{n^s}(s \in C \text{ and } Rs(s) > 0 \wedge (s \neq 1))$ and $\eta(s)=(1-2^{1-s})\zeta(s)(s \in C \text{ and } Rs(s) > 0 \wedge s \neq$

1, ζ(s) is the Riemann Zeta function) ,so Riemann $\zeta(s) = \frac{\eta(s)}{(1-2^{1-s})} = \frac{1}{(1-2^{1-s})}\sum_1^\infty \frac{(-1)^{n-1}}{n^s} = \frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-s})^{-1}$ $(s \in C \text{ and } Rs(s) > 0 \wedge (s \neq 1)$,n∈ Z_+, p ∈ Z_+, s ∈C, n goes through all the natural numbers, p goes through all the prime numbers). Let's prove that ζ(s) and $\zeta((\bar{s})$ are complex conjugations of each other.

$\sum_1^\infty \frac{(-1)^{n-1}}{n^s}$=[$1^{-\rho}$Cos(yln1)− $2^{-\rho}$Cos(yln2)+ $3^{-\rho}$Cos(yln3)−$4^{-\rho}$Cos(yln4)-...]-i[$1^{-\rho}$Sin(yln1) − $2^{-\rho}$sin(yln2)+ $3^{-\rho}$sin(yln3) − $4^{-\rho}$sin(yln4)+...]= U-Vi,

$\sum_1^\infty \frac{(-1)^{n-1}}{n^{\bar{s}}}$=[$1^{-\rho}$Cos(yln1)$-2^{-\rho}$Cos(yln2)+ $3^{-\rho}$Cos(yln3)$-4^{-\rho}$Cos(yln4)-...]+i[$1^{-\rho}$Sin(yln1) $-$ $2^{-\rho}$sin(yln2)+ $3^{-\rho}$sin(yln3)$-4^{-\rho}$sin(yln4)+...]= U+Vi,

$\sum_1^\infty \frac{(-1)^{n-1}}{n^{1-s}}$=[$1^{\rho-1}$Cos(yln1)$-$ $2^{\rho-1}$Cos(yln2)+ $3^{\rho-1}$Cos(yln3)$-4^{-\rho}$Cos(yln4)-...]+i[$1^{-\rho}$Sin(yln1)$-$ $2^{-\rho}$sin(yln2)+ $3^{-\rho}$sin(yln3) $-$ $4^{-\rho}$sin(yln4)+...],

$\sum_1^\infty \frac{(-1)^{n-1}}{n^{k-s}}$=[$1^{\rho-k}$Cos(yln1)$-$ $2^{\rho-k}$Cos(yln2)+ $3^{\rho-k}$Cos(yln3)$-4^{\rho-k}$Cos(yln4)-...]+i[$1^{\rho-k}$Sin(yln1)$-$ $2^{\rho-k}$sin(yln2)+ $3^{\rho-k}$sin(yln3) $-$ $3^{\rho-k}$sin(yln4)+...],

So

$\prod_p(1-p^{-s})^{-1}=\overline{\prod_p(1-p^{-\bar{s}})^{-1}}$,

So

$\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{n^s} = \overline{\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{n^{\bar{s}}}}$,

$\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-\bar{s}})^{-1}=\overline{\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-\bar{s}})^{-1}}$,

$\zeta(s)=\frac{1}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{n^s} = \frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-s})^{-1}$,

$\zeta(\bar{s})=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-\bar{s}})^{-1} = \frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-\bar{s}})^{-1}$.

So

Only $\zeta(s)=\overline{\zeta(\bar{s})}$ [2] .

Becuase

so

p^{1-s}=$p^{(1-\rho-yi)}$=$p^{1-\rho}x^{-yi}$=$p^{1-\rho}(\cos(\ln p)+i\sin(\ln p))^{-y}$=$p^{1-\rho}(\cos(y\ln p)-i\sin(y\ln p))$,

$p^{1-\bar{s}}=p^{(1-\rho+yi)}=p^{1-\rho}p^{yi}=p^{1-\rho}(p^{yi}) = p^{1-\rho}(\cos(\ln p)+i\sin(\ln p))^y=(p^{1-\rho}(\cos(y\ln p)+i\sin(y\ln p))$

Then

$p^{-(1-s)}$==$p^{(-1+\rho+yi)}$=$p^{\rho-1}x^{yi} = p^{\rho-1}\frac{1}{(\cos(y\ln p)-i\sin(y\ln p))}$ =$(p^{\rho-1}(\cos(y\ln p)+i\sin(y\ln p))$,

$p^{-(\bar{s})}$=$p^{-(\rho-yi)}$=$p^{-\rho}p^{yi} = (p^{-\rho}(\cos(y\ln p)+i\sin(y\ln p))$,

so

$(1-p^{-(1-s)})$=1-$(p^{\rho-1}(\cos(y\ln p)+i\sin(y\ln p))$ =$1-p^{\rho-1}\cos(y\ln p)-ip^{\rho-1}\sin(y\ln p)$,

$(1-p^{-(\bar{s})})$=1-$(p^{-\rho}(\cos(y\ln p)+i\sin(y\ln p))$ =$1-p^{-\rho}\cos(y\ln p)-ip^{-\rho}\sin(y\ln p)$,

$\sum_1^\infty \frac{(-1)^{n-1}}{n^{1-s}}$ =[$1^{\rho-1}$Cos(yln1)$-$ $2^{\rho-1}$Cos(yln2)+ $3^{\rho-1}$Cos(yln3)$-4^{\rho-1}$Cos(yln4)-...]+i[$1^{\rho-1}$Sin(yln1)$-$ $2^{\rho-1}$sin(yln2)+ $3^{\rho-1}$sin(yln3) $-$ $4^{\rho-1}$sin(yln4)+...] ,

$\sum_1^\infty \frac{(-1)^{n-1}}{n^{\bar{s}}}$ = [$1^{-\rho}$ Cos(yln1) $-2^{-\rho}$ Cos(yln2)+ $3^{-\rho}$ Cos(yln3) $-4^{-\rho}$ Cos(yln4)-...]+i[$1^{-\rho}$ Sin(yln1) $-$ $2^{-\rho}$sin(yln2)+ $3^{-\rho}$sin(yln3)$-4^{-\rho}$sin(yln4)+...], when $\rho=\frac{1}{2}$,

then

$\sum_1^\infty \frac{(-1)^{n-1}}{n^{1-s}}=\sum_1^\infty \frac{(-1)^{n-1}}{n^{\bar{s}}}$,

$(1-p^{-(1-s)})=(1-p^{-\bar{s}})$,

and

$(1-p^{-(1-s)})^{-1}=(1-p^{-\bar{s}})^{-1}$,

$\prod_p(1-p^{-(1-s)})^{-1}=\prod_p(1-p^{-\bar{s}})^{-1}$,

and

$\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-(1-s)})^{-1}=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-\bar{s}})^{-1}$,

$\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty \frac{(-1)^{n-1}}{n^{1-s}}=\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty \frac{(-1)^{n-1}}{n^{\bar{s}}}$,

and

$\zeta(1-s)=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-(1-s)})^{-1}$,

$\zeta(\bar{s})=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-\bar{s}})^{-1}$,

$\zeta(1-s)=\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty \frac{(-1)^{n-1}}{n^{1-s}}$,

$\zeta(\bar{s})=\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty \frac{(-1)^{n-1}}{n^{\bar{s}}}$,

so when $\rho=\frac{1}{2}$, then

Only $\zeta(1-s)=\zeta(\bar{s})$.

According the equation $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ obtained by Riemann,since Riemann has shown that the Riemann $\zeta(s)$ function has zero, that is, in

$\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$,$\zeta(s)=0$ is true.

When $\zeta(s)=0$, then only $\zeta(k-\bar{s})=\zeta(s)=0$, and

When $\zeta(\bar{s})=0$,then $\zeta(k-s)=\zeta(\bar{s})=0$.

But the Riemann $\zeta(s)$function only satisfies $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$,so when $\zeta(s)=0$, then only $\zeta(1-s)=\zeta(s)=0$, and when $\zeta(\bar{s})=0$, then only $\zeta(1-s)=\zeta(\bar{s})=0$, which is $\zeta(k-s)=\zeta(1-s)=\zeta(\bar{s})$,so only k=1 be true.so only $\mathrm{Re}(s)=\frac{k}{2}=\frac{1}{2}$ is true .

$\sum_1^\infty \frac{(-1)^{n-1}}{n^{k-s}}=[\,1^{\rho-k}\mathrm{Cos}(y\ln 1)-2^{\rho-k}\mathrm{Cos}(y\ln 2)+3^{\rho-k}\mathrm{Cos}(y\ln 3)-4^{\rho-k}\mathrm{Cos}(y\ln 4)-\ldots]+i[1^{\rho-k}\mathrm{Sin}$

$(y\ln 1) - 2^{\rho-k}\sin(y\ln 2) + 3^{\rho-k}\sin(y\ln 3) - 3^{\rho-k}\sin(y\ln 4) + \ldots]$,

$\sum_1^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}}$=[$1^{-\rho}$ Cos(yln1)$-2^{-\rho}$ Cos(yln2)+ $3^{-\rho}$Cos(yln3)$-4^{-\rho}$Cos(yln4)-...]+i[$1^{-\rho}$Sin(yln1)$-$ $2^{-\rho}$ sin(yln2)+ $3^{-\rho}$sin(yln3)$-4^{-\rho}$sin(yln4)+...],

$p^{k-s}=p^{(k-\rho-yi)}=p^{k-\rho}x^{-yi}=p^{k-\rho}(\cos(\ln p) + i\sin(\ln p))^{-y}=p^{k-\rho}(\cos(y\ln p) - i\sin(y\ln p))$,

$p^{1-\bar{s}}=p^{(1-\rho+yi)}=p^{1-\rho}p^{yi}=p^{1-\rho}(p^{yi}) = p^{1-\rho}(\cos(\ln p) + i\sin(\ln p))^{y}=(p^{1-\rho}(\cos(y\ln p) + i\sin(y\ln p))$,

Then

$p^{-(k-s)}=p^{(-k+\rho+yi)}=p^{\rho-k}x^{yi} = p^{\rho-k}\frac{1}{(\cos(y\ln p)-i\sin(y\ln p))}=(p^{\rho-k}(\cos(y\ln p) + i\sin(y\ln p))$,

$p^{-(\bar{s})}=p^{-(\rho-yi)}=p^{-\rho}p^{yi} = (p^{-\rho}(\cos(y\ln p) + i\sin(y\ln p))$,

$p^{-(k-s)}=(p^{\rho-k}(\cos(y\ln p) + i\sin(y\ln p))$,

so

$(1 - p^{-(k-s)})=1-(p^{\rho-k}(\cos(y\ln p) + i\sin(y\ln p)) = 1 - p^{\rho-k}\cos(y\ln p) - ip^{\rho-k}\sin(y\ln p)$,

$(1 - p^{-\bar{s}})=1-(p^{-\rho}(\cos(y\ln p) + i\sin(y\ln p)) = 1 - p^{-\rho}\cos(y\ln p) - ip^{-\rho}\sin(y\ln p)$,

So

so when $\rho=\frac{k}{2}(k\in R)$ then

$\sum_1^{\infty}\frac{(-1)^{n-1}}{n^{k-s}}=\sum_1^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}}$,

$(1 - p^{-(k-s)}) = (1 - p^{-\bar{s}})$

and

$(1 - p^{-(k-s)})^{-1}=(1 - p^{-\bar{s}})^{-1}$,

$\prod_p(1 - p^{-(k-s)})^{-1}=\prod_p(1 - p^{-\bar{s}})^{-1}$,

and

$$\frac{1}{(1-2^{1-s})}\sum_1^{\infty}\frac{(-1)^{n-1}}{n^{k-s}} = \frac{1}{(1-2^{1-s})}\sum_1^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}},$$

and

$\zeta(k-s)=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1 - p^{-(k-s)})^{-1}$,

$\zeta(\bar{s})=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1 - p^{-\bar{s}})^{-1}$,

$\zeta(k-s)=\frac{1}{(1-2^{1-s})}\sum_1^{\infty}\frac{(-1)^{n-1}}{n^{k-s}}$,

$\zeta(\bar{s})=\frac{1}{(1-2^{1-s})}\sum_1^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}}$, so when $\rho=\frac{k}{2}(k\in R)$ then

Only $\zeta(k-s)=\zeta(\overline{s})$.

According the equation $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}Cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ obtained by Riemann,since Riemann has shown that the Riemann $\zeta(s)$ function has zero, that is, in

$\zeta(1\text{-}s)=2^{1-s}\pi^{-s}Cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$,$\zeta(s)=0$ is true.

When $\zeta(s)=0$, then only $\zeta(k-\overline{s})=\zeta(s)=0$, and
When $\zeta(\overline{s})=0$,then $\zeta(k-s)=\zeta(\overline{s})=0$.And because when $\zeta(\overline{s})=0$, then only $\zeta(1-s)=\zeta(\overline{s})=0$, which is $\zeta(k-s)==\zeta(\overline{s})$,so only k=1 be true.
According $\zeta(s)=\zeta(1\text{-}s)=0$ and $\zeta(s)=\zeta(\overline{s})=\zeta(1\text{-}\overline{s})=0$,then $s=\overline{s}$ or $s=1\text{-}s$ or $\overline{s}=1\text{-}s$,so $s\in R$, or $\rho+yi=1\text{-}\rho\text{-}yi$,or $\rho\text{-}yi=1\text{-}\rho\text{-}yi$, so $s\in R$, or $\rho=\frac{1}{2}$ and $y=0$,or $\rho=\frac{1}{2}$ and $y\in R$ and $y\neq 0$,so $s\in R$, for example $s=-2n(n\in Z_+)$, or $s=\frac{1}{2}+oi$,or $s=\frac{1}{2}+yi(y\in R \text{ and } y\neq 0)$.$\zeta(\frac{1}{2})>\zeta(1)=\infty$,drop it, $s=-2n(n\in Z_+)$,It's the trivial zero of the Riemann $\zeta(s)$ function, drop it.

Beacause only when $\rho=\frac{1}{2}$,the next three equations, $\zeta(\rho+yi)=0$, $\zeta(1-\rho-yi)=0$, and $\zeta(\rho\text{-}yi)=0$ are all true, $\zeta(\frac{1}{2})>\zeta(1)=\infty$, so only $s=\frac{1}{2}+yi$ $(y\in R \text{ and } y\neq 0, s\in C)$ is true,or say only $s=\frac{1}{2}+ti$ $(t\in R \text{ and } t\neq 0, s\in C)$ is true. Since Riemann has shown that the Riemann $\zeta(s)$ function has zero, that is, in $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}Cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$, $\zeta(s)=0$ is true.According the equation $\xi(s)=\frac{1}{2}s(s\text{-}1)\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ obtained by Riemann , so $\xi(s)=\xi(1-s)$, because $\Gamma(\frac{s}{2})=\overline{\Gamma(\frac{\overline{s}}{2})}$, and $\pi^{-\frac{s}{2}}=\overline{\pi^{-\frac{\overline{s}}{2}}}$, and because $\zeta(s)=\overline{\zeta(\overline{s})}$, so $\xi(s)=\overline{\xi(\overline{s})}$. So when $\zeta(s)=0$,then $\xi(s)=\zeta(1-s)=\zeta(\overline{s})=0$and $\xi(s)=\xi(1-s)=\xi(\overline{s})=0$ must be true , so the zeros of the Riemann $\zeta(s)$ function and the nontrivial zeros of the Riemann $\xi(s)$ function are identical, so the complex root of Riemann $\xi(s)=0$ satisfies $s=\frac{1}{2}+ti$ $(t\in R \text{ and } t\neq 0, s\in C)$, according to the Riemann function $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)$ and he Riemann hypothesis $s=\frac{1}{2}+ti$, because $s\neq 1$, and $\prod\frac{s}{2}\neq o$, $\pi^{-\frac{s}{2}}\neq o$, so $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\neq o$, and when $\xi(t)=0$, then $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$, and $\zeta(\frac{1}{2}+ti)=\frac{\xi(t)}{\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}}=\frac{0}{\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}}=0$, so $t\in R$ and $t\neq 0$. So the root t of the equations $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$ and $4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx=\xi(t)=0$ and $\xi(t)=\frac{1}{2}\text{-}(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\,x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)=0$ must be real and $t \neq 0$ .If Re(s)= $\frac{k}{2}(k \in R)$,then $\zeta(k\text{-}s)= 2^{k-s}\pi^{-s}\,Cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ and $\xi(k-s) = \frac{1}{2}s(s\text{-}k)\,\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ are true , so when $\zeta(s)=0$,then $\zeta(s) = \zeta(k-s) = \zeta(\overline{s}) = 0$ and $\xi(s)=\xi(k-s)=\xi(\overline{s})=0$ must be true , and $s=\frac{k}{2}+ti$ (k∈R, t∈R and $t \neq 0$,s∈C) must be true, then $\prod\frac{s}{2}(s\text{-}k)\pi^{-\frac{s}{2}}\zeta(\frac{k}{2}+ti)=\xi(t)=0$, and $\zeta(\frac{k}{2}+ti)=\frac{\xi(t)}{\prod\frac{s}{2}(s-k)\pi^{-\frac{s}{2}}}=\frac{0}{\prod\frac{s}{2}(s-k)\pi^{-\frac{s}{2}}}=0$,so t∈R and $t \neq 0$. So the root t of the equations $\prod\frac{s}{2}(s\text{-}k)\pi^{-\frac{s}{2}}\zeta(\frac{k}{2}+ti)=\xi(t)=0$ must be real and $t \neq 0$. But the Riemann $\zeta(s)$function only satisfies $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}Cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ and $\xi(s) = \frac{1}{2}s(s\text{-}1)\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$, is also say that only $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}Cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ is true , so only Re(s)=$\frac{k}{2}=\frac{1}{2}$ is true, so only k=1 is true.The Riemann hypothesis and the Riemann conjecture must satisfy the properties of the Riemann $\zeta(s)$ function and the Riemann $\xi(s)$ function, The properties of the Riemann $\zeta(s)$ function and the Riemann $\xi(s)$ function are fundamental, the Riemann hypothesis and the Riemann conjecture must be correct to reflect the properties of the Riemann $\zeta(s)$ function and the Riemann $\xi(s)$ function, that is, the roots of the Riemann $\xi(t)$ function can only be real, that is, Re(s) can only be equal to $\frac{1}{2}$, and Im(s) must be real, and Im(s) is not equal to zero.So the Riemann hypothesis and the Riemann conjecture must be correct.

For any complex number s, When Rs(s) is any real number, including $Rs(s)>0 \wedge (s \neq 1)$ and $Rs(s) \leq 0 \wedge (s \neq 0)$, then Riemann Zeta() function is $\zeta(s)= 2^s\pi^{s-1}\,Sin(\frac{\pi s}{2})\Gamma(1\text{-}s)\zeta(1\text{-}s)$.

Suppose s=ρ+yi (ρ∈R,y∈R and $y \neq 0$,s∈C),let's prove that $\zeta(s)$ and $\zeta(\overline{s})$ are complex conjugations of each other and get the equation $\zeta(s)=2^s\pi^{s-1}Sin(\frac{\pi s}{2})\Gamma(1\text{-}s)\zeta(1\text{-}s)$.

The reasoning in Riemann's paper goes like:

$2\sin(\pi s)\prod(s-1)\zeta(s)=(2\pi)^s\sum n^{s-1}((-i)^{s-1}+i^{s-1})$ [1] (Formula 3),

based on euler's $e^{ix}=\cos(x)+i\sin(x)$ $(x \in R)$ can get

$e^{i(-\frac{\pi}{2})}=\cos(\frac{-\pi}{2}) + i\sin(\frac{-\pi}{2}) = 0\text{-}i = \text{-}i$,

$e^{i(\frac{\pi}{2})}=\cos(\frac{\pi}{2})+i\sin(\frac{\pi}{2})=0+i=i$,

then

$(-i)^{s-1}+i^{s-1} = (-i)^{-1}(-i)^s+(i)^{-1}(i)^s=(-i)^{-1}e^{i(-\frac{\pi}{2})s}+i^{(-1)}e^{i(\frac{\pi}{2})s}=$

$ie^{i(-\frac{\pi}{2})s}\text{-}ie^{i(\frac{\pi}{2})s} = i(\cos\frac{-\pi s}{2}+i\sin\frac{-\pi s}{2})\text{-}i(\cos\frac{\pi s}{2}+i\sin\frac{\pi s}{2})=i\cos(\frac{\pi s}{2})\text{-}i\cos(\frac{\pi s}{2})+\sin(\frac{\pi s}{2})+\sin(\frac{\pi s}{2})$

$=2\sin(\frac{\pi s}{2})$ (Formula 4)

According to the property of $\Pi(s-1)=\Gamma(s)$ of the gamma function,and
$\Sigma n^{s-1}=\zeta(1-s)$,
Substitute the above (Formula 4) into the above (Formula 3), will get

$2\sin(\pi s)\Gamma(s)\zeta(s)=(2\pi)^s\zeta(1-s)2\sin\frac{\pi s}{2}$ (Formula 5),

If I substitute it into (Formula5), according to the double Angle formula $\sin(\pi s)=2\mathrm{Sin}(\frac{\pi s}{2})\mathrm{Cos}(\frac{\pi s}{2})$,

we Will get $\zeta(1-s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ (Formula 6),

When $s\neq -2n(n\in Z_+)$,because $\pi^{-\frac{1-s}{2}}\neq 0\neq 0$ and $\Gamma(\frac{1-s}{2})\neq 0$,so when $\zeta(s)=0$, then $\zeta(1-s)=0$,

Substituting s→1-s, that is taking s as 1-s into Formula 6, we will get

$\zeta(s)=2^s\pi^{s-1}\mathrm{Sin}(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)$ (Formula 7),

This is the functional equation for $\zeta(s)$. To rewrite it in a symmetric form, use the residual formula of the gamma function [3]

$\Gamma(Z)\Gamma(1-Z)=\frac{\pi}{\sin(\pi Z)}$ (Formula 8)

and Legendre's formula

$\Gamma(\frac{Z}{2})\Gamma(\frac{Z}{2}+\frac{1}{2})=2^{1-Z}\pi^{\frac{1}{2}}\Gamma(Z)$ (Formula 9) ,

Take $z=\frac{s}{2}$ in (Formula 8) and substitute it to get

$\sin(\frac{\pi s}{2})=\frac{\pi}{\Gamma(\frac{s}{2})\Gamma(1-\frac{s}{2})}$ (Formula 10) ,

In (Formula 9), let z=1-s and substitute it in to get

$\Gamma(1-s)=2^{-s}\pi^{-\frac{1}{2}}\Gamma(\frac{1-s}{2})\Gamma(1-\frac{s}{2})$ (Formula 11)

By substituting (Formula 10) and (Formula 11) into (Formula 7), we get

$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)$,

also

$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ is invariant under the transformation s→1-s,

And that's exactly what Riemann said in his paper.
That is to say:

$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ is invariant under the transformation s→1-s ,

also

$\prod(\frac{s}{2}-1)\pi^{-\frac{s}{2}}\zeta(s)=\prod(\frac{1-s}{2}-1)\pi^{-\frac{1-s}{2}}\zeta(1-s)$

or

$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)$ (Formula 2),

Then $\zeta(s)=2^s\pi^{s-1}\mathrm{Sin}(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)$,

under the transformation s→1-s ,will get

$\zeta(1-s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ (Formula 1)

Because $L(s, \chi(n))=\chi(n)\zeta(s)$ and $L(1-s, \chi(n))=\chi(n)\zeta(1-s)$,

and according to $\zeta(s)=2^s\pi^{s-1}\mathrm{Sin}(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)$ (Formula 7),

so

Only $L(s, \chi(n))=2^s\pi^{s-1}\mathrm{Sin}(\frac{\pi s}{2})\Gamma(1-s)L(1-s, \chi(n))$(Formula 12).

According to the property that Gamma function Γ(s) and exponential function are nonzero, is also that $\Gamma(\frac{1-s}{2})\neq 0$, and $\pi^{-\frac{1-s}{2}}\neq 0$, according to $\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)$ (Formula 2),

Mathematicians have shown that the real part of the complex independent variable s of the Riemann ζ(s)function will have zero only if 0<Re(s)<1 and Im(s)≠ 0, so we agree on Riemann $\zeta(s) = \frac{\eta(s)}{(1-2^{1-s})} = \frac{1}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{n^s} = \frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-s})^{-1}$ ($s \in C$ and $0 < Rs(s) < 1 \wedge (s \neq 1)$ and $Im(s) \neq 0$,n$\in Z_+, p \in Z_+, s \in C$, n goes through all the natural numbers, p goes through all the prime numbers).

According the equation $\zeta(1-s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ obtained by Riemann,since Riemann has shown that the Riemann ζ(s) function has zero, that is, in $\zeta(1-s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$, so ζ(s)=0 is true, and so we agree on $\zeta(1-s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$($s \in C$ and $0 < Rs(s) < 1 \wedge (s \neq 1)$ and $Im(s) \neq 0$,n$\in Z_+, p \in Z_+, s \in C$, n goes through all the natural numbers, p goes through all the prime numbers).

According to the property that Gamma function Γ(s) and exponential function are nonzero, is also that $\Gamma(\frac{1-s}{2})\neq 0$, and $\pi^{-\frac{1-s}{2}}\neq 0$,

So when ζ(s)=0, then ζ(1-s)=0, also must ζ(s)=ζ(1-s)=0.

Because $\sin(Z)=\frac{e^{iZ}-e^{-iZ}}{2i}$,Suppose Z=s=ρ+yi (ρ∈R,y∈R and $y \neq 0$,s∈C), then

$$\sin(s)=\frac{e^{is}-e^{-is}}{2i} = \frac{e^{i(\rho+yi)}-e^{-i(\rho+yi)}}{2i},$$

$$\sin(\bar{s})=\frac{e^{i\bar{s}}-e^{-i\bar{s}}}{2i} = \frac{e^{i(\rho-yi)}-e^{-i(\rho-yi)}}{2i},$$

according $x^{s}=x^{(\rho+yi)}=x^{\rho}x^{yi}=x^{\rho}(\cos(\ln x)+i\sin(\ln x))^{y}=x^{\rho}(\cos(y\ln x)+i\sin(y\ln x))$, then
$e^{s}=e^{(\rho+yi)}=e^{\rho}e^{yi}=e^{\rho}(\cos(y)+i\sin(y)) = e^{\rho}(\cos(y)+i\sin(y))$,
$e^{is}=e^{i(\rho+yi)}=e^{\rho i}(\cos(iy)+i\sin(iy)) = (\cos(\rho)+i\sin(\rho))(\cos(iy)+i\sin(iy))$
$e^{i\bar{s}} = e^{i(\rho-yi)}=e^{\rho i}(\cos(-iy)+i\sin(-iy)) = (\cos(\rho)+i\sin(\rho))(\cos(iy)-i\sin(iy))$,
$e^{-is}=e^{-i(\rho+yi)}=e^{-\rho i}(\cos(-iy)+i\sin(-iy)) = (\cos(\rho)-i\sin(\rho))(\cos(iy)-i\sin(iy))$
$e^{-i\bar{s}} = e^{-i(\rho-yi)}=e^{-\rho i}(\cos(iy)+i\sin(iy)) = (\cos(\rho)-i\sin(\rho))(\cos(iy)+i\sin(iy))$,
$2^{s}=2^{(\rho+yi)}=2^{\rho}2^{yi}=2^{\rho}(\cos(\ln 2)+i\sin(\ln 2))^{y}=2^{\rho}(\cos(y\ln 2)+i\sin(y\ln 2))$,
$2^{\bar{s}}=2^{(\rho-yi)}=2^{\rho}2^{-yi}=2^{\rho}(\cos(\ln 2)+i\sin(\ln 2))^{-y}=2^{\rho}(\cos(y\ln 2)-i\sin(y\ln 2))$,
$\pi^{s-1}=2^{(\rho-1+yi)}=2^{\rho-1}2^{yi}=2^{\rho-1}(\cos(\ln 2)+i\sin(\ln 2))^{y}=2^{\rho-1}(\cos(y\ln 2)+i\sin(y\ln 2))$,
$\pi^{\bar{s}-1}=2^{(\rho-1-yi)}=2^{\rho-1}2^{-yi}=2^{\rho}(\cos(\ln 2)+i\sin(\ln 2))^{-y}=2^{\rho-1}(\cos(y\ln 2)-i\sin(y\ln 2))$,
So

$2^{s}=\overline{2^{\bar{s}}}$, $\pi^{s-1}=\overline{\pi^{\bar{s}-1}}$,

and

$\frac{e^{is}-e^{-is}}{2i}=\overline{\frac{e^{i\bar{s}}-e^{-i\bar{s}}}{2i}}$,

So
$\mathrm{Sin}(s)=\overline{\mathrm{Sin}(\bar{s})}$,
So

$\mathrm{Sin}(\frac{\pi s}{2})=\overline{\mathrm{Sin}(\frac{\pi\bar{s}}{2})}$.

And the gamma function on the complex field is defined as:

$\Gamma(s)=\int_{0}^{+\infty} t^{s-1}e^{-t}dt$

among
Re(s)>0,this definition can be extended by the analytical continuation principle to the entire field of complex numbers, except for non-positive integers,
So
$\Gamma(s)=\overline{\Gamma(\bar{s})}$,
and
$\Gamma(1\text{-}s)=\overline{\Gamma(1-\bar{s})}$.

When $\zeta(1\text{-}\bar{s})=\overline{\zeta(1-\bar{s})}=0=\zeta(s)=\zeta(1\text{-}s)=0$, and according $\zeta(s)=2^{s}\pi^{s-1}\mathrm{Sin}(\frac{\pi s}{2})\Gamma(1\text{-}s)\zeta(1\text{-}s)$, then

Only $\zeta(s)=\overline{\zeta(\bar{s})}=0$,is also say $\zeta(s)=\zeta(\bar{s})=\zeta(1\text{-}\bar{s})=0$. so only $\zeta(\rho+yi)=\zeta(\rho\text{-}yi)=0$ is true.

According the equation $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ obtained by Riemann,since Riemann has shown that the Riemann $\zeta(s)$ function has zero, that is, in $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$, $\zeta(s)=0$ is true, so when $\zeta(s)=0$, then only $\zeta(s)=\zeta(1\text{-}s)=0$ is true.
According $\zeta(s)=\zeta(1\text{-}s)=0$ and $\zeta(s)=\zeta(\bar{s})=\zeta(1\text{-}s)=0$,then only $s=\bar{s}$or $s=1\text{-}s$ or $\bar{s}=1\text{-}s$,so $s\in R$, or $\rho+yi=1\text{-}\rho\text{-}yi$,or $\rho\text{-}yi=1\text{-}\rho\text{-}yi$, so $s\in R$, or $\rho=\frac{1}{2}$and $y=0$,or $\rho=\frac{1}{2}$ and $y\in R$ and $y\neq$

0,so $s \in R$, for example s=-2n($n\in Z_+$), or s=$\frac{1}{2}$+oi ,or s=$\frac{1}{2}$+yi(y $\in$R and y $\neq$ 0).$\zeta(\frac{1}{2}) > \zeta(1) = \infty$,drop it, s=-2n($n\in Z_+$),It's the trivial zero of the Riemann $\zeta(s)$ function, drop it.

Beacause only when $\rho=\frac{1}{2}$,the next three equations, $\zeta(\rho+yi)$=0, $\zeta(1-\rho-yi)$=0, and $\zeta(\rho\text{-}yi)$=0 are all true, $\zeta(\frac{1}{2}) > \zeta(1) = \infty$,so only s=$\frac{1}{2}$+yi (y$\in$R and y $\neq$ 0,s$\in$C) is true,or say only s=$\frac{1}{2}$+ti (t$\in$R and t $\neq$ 0,s$\in$C)is true.Since Riemann has shown that the Riemann $\zeta(s)$ function has zero, that is, in $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$, $\zeta(s)$=0 is true.According the equation$\xi(s) = \frac{1}{2}s(s\text{-}1)\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ obtained by Riemann, so $\xi(s)=\xi(1-s)$, because $\Gamma(\frac{s}{2})=\overline{\Gamma(\frac{\bar{s}}{2})}$, and $\pi^{-\frac{s}{2}}=\overline{\pi^{-\frac{\bar{s}}{2}}}$, and because $\zeta(s)=\overline{\zeta(\bar{s})}$, so $\xi(s)=\overline{\xi(\bar{s})}$. So when $\zeta(s)$=0 ,then $\xi(s)=\zeta(1-s) = \zeta(\bar{s}) = 0$and $\xi(s)=\xi(1-s)=\xi(\bar{s})$=0 must be true , so the zeros of the Riemann $\zeta(s)$ function and the nontrivial zeros of the Riemann $\xi(s)$ function are identical, so the complex root of Riemann $\xi(s)$=0 satisfies s=$\frac{1}{2}$+ti (t$\in$R and t $\neq$ 0,s$\in$C) , according to the Riemann function $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\zeta(s\ )=\xi(t)$ and he Riemann hypothesis s=$\frac{1}{2}$+ti, because s$\neq$1, and $\prod\frac{s}{2}\neq$o, $\pi^{-\frac{s}{2}}\neq$o, so $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\neq$o, and when $\xi(t)$=0, then $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$, and $\zeta(\frac{1}{2}+ti)=\frac{\xi(t)}{\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}}=\frac{0}{\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}}=0$, so t$\in$R and t $\neq$ 0. So the root t of the equations $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti\ )=\xi(t)=0$ and $4\int_1^\infty\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx=\xi(t)=0$ and $\xi(t)=\frac{1}{2}\text{-}\ (t^2+\frac{1}{4})\int_1^\infty\Psi(x)\ x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x\ )=0$ must be real and $t\neq0$ .If Re(s)= $\frac{k}{2}(k\in R)$,then $\zeta(k\text{-}s)=\ 2^{k-s}\pi^{-s}\ \mathrm{Cos}(\ \frac{\pi s}{2}\ )\Gamma(s)\zeta(s)$ and $\xi(k-s) = \frac{1}{2}s(s\text{-}k)\,\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ are true , so when $\zeta(s)$=0 ,then $\zeta(s) = \zeta(k-s) = \zeta(\bar{s}) = 0$ and $\xi(s)=\xi(k-s)=\xi(\bar{s})$=0 must be true , and s=$\frac{k}{2}$+ti (k$\in$R, t$\in$R and t $\neq$ 0,s$\in$C) must be true, then $\prod\frac{s}{2}(s\text{-}k)\pi^{-\frac{s}{2}}\zeta(\frac{k}{2}+ti)=\xi(t)=0$, and $\zeta(\frac{k}{2}+ti)=\frac{\xi(t)}{\prod\frac{s}{2}(s-k)\pi^{-\frac{s}{2}}}=\frac{0}{\prod\frac{s}{2}(s-k)\pi^{-\frac{s}{2}}}=0$,so t$\in$R and t $\neq$ 0. So the root of the equations $\prod\frac{s}{2}(s\text{-}k)\pi^{-\frac{s}{2}}\zeta(\frac{k}{2}+ti\ )=\xi(t)=0$ must be real and $t\neq0$. But the Riemann $\zeta(s)$function only satisfies $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ and $\xi(s) = \frac{1}{2}s(s\text{-}1)\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$, is also say that only $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ is true , so only $\mathrm{Re}(s)=\frac{k}{2}=\frac{1}{2}$ is true, so only k=1 is true.The Riemann hypothesis and the Riemann conjecture must satisfy the properties of the Riemann $\zeta(s)$ function and the Riemann $\xi(s)$ function, The properties of the Riemann $\zeta(s)$ function and the Riemann $\xi(s)$ function are fundamental, the Riemann hypothesis and the Riemann conjecture must be correct to reflect the properties of the Riemann $\zeta(s)$ function and the Riemann $\xi(s)$ function, that is, the roots of the Riemann $\xi(t)$ function can only be real, that is, Re(s) can only be equal to $\frac{1}{2}$, and Im(s) must be real, and Im(s) is not equal to zero.So the Riemann hypothesis and the Riemann conjecture must be correct.

Riemann found in his paper that

$$\prod\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}\zeta(s) = \int_1^{\infty}\psi(x)\ x^{\frac{s}{2}-1}dx+\int_1^{\infty}\psi(\frac{1}{x})\ x^{\frac{s-3}{2}}dx$$

$$+\frac{1}{2}\int_0^1(x^{\frac{s-3}{2}}\ -x^{\frac{s}{2}-1})dx$$

$$=\frac{1}{s(s-1)}+\int_1^{\infty}\psi(x)\ (x^{\frac{s}{2}-1}+x^{-\frac{1+s}{2}})dx\ ,$$

Because $\frac{1}{s(s-1)}$ and $\int_1^{\infty}\psi(x)\ (x^{\frac{s}{2}-1}+x^{-\frac{1+s}{2}})dx$ are all invariant under the transformation s→1-s If I introduce the auxiliary function $\psi(s)=\prod\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}\zeta(s)$,

So I can just write it as $\psi(s)=\psi(1\text{-}s)$. But it would be more convenient to add the factor $s(s-1)$to $\psi(s)$ and introduce the coefficient $\frac{1}{2}$, which is exactly what Riemann did, is that to take$\xi(s)=\frac{1}{2}s(s\text{-}1)\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}$ $\zeta(s)$.Because the factor (s-1) cancels out the first pole of $\zeta(s)$at s=1, And the factor s cancels out the pole of $\Gamma(\frac{s}{2})$ at s=0, and s is equal to -2, -4, -6,...,the rest of the poles of $\Gamma(\frac{s}{2})$ cancel out . So $\xi(s)$ is an integral function.And notice that the factor $s(s-1)$ obviously doesn't change under the transformation $s\rightarrow 1-s$,So we also have the function $\xi(s)=\xi(1-s)$. Base on $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ (Formula 1).When $\sin(\frac{\pi s}{2})=0$, then if s= -2n($n\in Z_+$), $\xi(s)$ is going to take the zero . At the same time, according to $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\mathrm{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$,when $s\neq 1+2n(n\in Z_+)$, and if $\zeta(s)=0$,then must $\zeta(1\text{-}s)=0$, is that to say $\zeta(s)=\zeta(1\text{-}s)=0$. According to Riemann's hypothesis $s=\frac{1}{2}+ti(t\in C,s\in C \text{ and } t\neq 0)$, s and t differ by a linear transformation . It's a 90 degree rotation plus a translation of $\frac{1}{2}$. So line $\mathrm{Re}(s)=\frac{1}{2}$ in the s plane corresponds to the real number line in the t plane,the zero of Riemann $\zeta(s)$ on the critical line $\mathrm{Re}(s)=\frac{1}{2}$ corresponds to the real root of $\xi(t)(t\in C \text{ and } t\neq 0)$.In Riemann function$\xi(t)$, the function equation $\xi(s)=\xi(1-s)$ becomes equation $\xi(t)=\xi(-t)$. $\xi(t)(t\in C \text{ and } t\neq 0)$ is an even function, an even function is a symmetric function, it's zeros are distributed symmetrically with respect to t=0 .The function$\xi(t)(t\in C, \text{and } t\neq 0)$ designed by Riemann and

Riemann's hypothesis $s=\frac{1}{2}+ti(t\in C, s\in C, \text{and } t\neq 0)$ and $\xi(s)=\xi(1-s)$ are equivalent to $\xi(t)$=$\xi(-t)$.So the function $\xi(s)$ is also an even function.The zero points on the graph of an even function $\xi(s)$ with respect to the coordinates of its argument on the real number line equal to some value are symmetrically distributed on the line perpendicular to the real number line of the complex plane. When $\xi(t)$=0, is also that $\xi(t)$=$\xi(-t)$=0,the zeros of $\xi(t)$ are symmetrically distributed with respect to t equals 0.When $\xi(s)$=0,is also that $\xi(s)$=$\xi(1-s)$=0,the zeros of $\xi(s)$ are symmetrically distributed with respect to point ($\frac{1}{2}$,0i) on a line perpendicular to the real number line of the complex plane.So when $\xi(s)$=$\xi(1-s)$=0, s and 1-s are pair of zeros of the function $\xi(s)$ symmetrically distributed in the complex plane with respect to point ($\frac{1}{2}$,0i) on a line perpendicular to the real number line of the complex plane.When $\zeta(s)$=0, $\zeta(1\text{-}s)$=0 is aslo that $\zeta(s)$=$\zeta(1\text{-}s)$=0. We find $\zeta(s)$=$\zeta(1\text{-}s)$=0 and $\xi(s)$=$\xi(1-s)$=0 are just the name of the function is idifferent,the independent variable s is equal to $\frac{1}{2}$+ti(t∈C,s∈C),that means that the zero arguments of function $\zeta(s)$ and function $\xi(s)$ are exactly the same,so the zeros of the $\zeta(s)$ function in the complex plane also correspond to the symmetric distribution of point ($\frac{1}{2}$,0i) on a line perpendicular to the real number line in the complex plane,so When $\zeta(s)=\zeta(1-s)=0$,s and 1-s are pair of zeros of the function $\zeta(s)$ symmetrically distributed in the complex plane with respect to point ($\frac{1}{2}$,0i) on a line perpendicular to the real number line of the complex plane.We got $\overline{\zeta(s)}$ =$\zeta(\bar{s})$(s=ρ+yi,$\rho\in R$, y ∈R and $y\neq 0$)

before,When t in Riemann's hypothesis s=$\frac{1}{2}$+ti(t∈C,s∈C and $t\neq 0$) is a complex number, and s=$\frac{1}{2}$+ti=ρ+yi , then s in $\overline{\zeta(s)}$=$\zeta(\bar{s})$(s=ρ+yi,$\rho\in R$, y ∈R and $y\neq 0$) is consistent with s in Riemann's hypothesis s=$\frac{1}{2}$+ti(t∈C,s∈C andt $\neq 0$).If $\zeta(s)$=$\zeta(\bar{s})$=0(s=ρ+yi,$\rho\in R$, y ∈R and $y\neq 0$),Since s and $\bar{s}$ are a pair of conjugate complex numbers,So s and $\bar{s}$ must be a pair of zeros of the function $\zeta(s)$ in the complex plane with respect to point (ρ,0i) on a line perpendicular to the real number line.s is a symmetric zero of 1-s, and a symmetric zero of $\bar{s}$. By the definition of complex numbers, how can a symmetric zero of the same function $\zeta(s)$ of the same zero independent variable s on a line perpendicular to the real number axis of the complex plane be both a symmetric zero of 1-s on a line perpendicular to the real number axis of the complex plane with respect to point ($\frac{1}{2}$,0i) and a symmetric zero of $\bar{s}$ on a line perpendicular to the real number axis of the complex plane with respect to point (ρ,0i)?Unless ρ and $\frac{1}{2}$ are the same value, is also that $\rho=\frac{1}{2}$, and only 1-s=$\bar{s}$ is true, and 1-s=s is wrong.Otherwise it's impossible,this is determined by the uniqueness of the zero of the function $\zeta(s)$ on the line passing through that point perpendicular to the real number axis of the complex plane with respect to the vertical foot symmetric distribution of the zero of the line and the real number axis of the complex plane,Only one line can be drawn perpendicular from the zero independent variable s of the function $\zeta(s)$ to the real number line of the complex plane, the vertical line has only one point of intersection with the real number axis of the complex plane. In the same complex plane, the same zero point of the function $\zeta(s)$ on the line passing through that point perpendicular to the real number line of the complex plane there will be only one zero point about the vertical foot symmetric distribution of the line and the real number line of the complex plane.Because $\overline{\zeta(s)}=\zeta(\bar{s})$(s= ρ +yi, $\rho \in R$, $y \in R$ and $y \neq 0$), then if $\zeta(\rho+yi)$=0, then $\zeta(\rho-yi)$=0, and because $\zeta(s)=\zeta(1-s)$=0, then $\zeta(1-\rho-yi)$=0, and because $\zeta(s)=\zeta(1-s)$=0, then $\zeta(1-\rho-yi)$=0. The next three equations, $\zeta(\rho+yi)$=0, $\zeta(\rho-yi)$=0, and $\zeta(1-\rho-yi)$=0, are all true, so only 1-ρ=ρ is true,only $s=\frac{1}{2}+ti$ (t∈R and $t \neq 0$,s∈C) is true.Since the harmonic series $\zeta(1)$ diverges, it has been proved by the late medieval French scholar Orem (1323-1382).The Riemann hypothesis and the Riemann conjecture must satisfy the properties of the Riemann $\zeta(s)$ function and the Riemann $\xi(s)$ function, The properties of the Riemann $\zeta(s)$ function and the Riemann $\xi(s)$ function are fundamental, the Riemann hypothesis and the Riemann conjecture must be correct to reflect the properties of the Riemann $\zeta(s)$ function and the Riemann $\xi(s)$ function, that is, the roots of the Riemann $\xi(t)$ function must only be real, that is, Re(s) can only be equal to $\frac{1}{2}$, and Im(s) must be real, and Im(s) is not equal to zero.So the Riemann hypothesis and the Riemann conjecture must be correct.

Riemann got $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)$ and $\xi(t)=\frac{1}{2}-(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\ x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)\,dx$ in his paper,or

$\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)$ and $\xi(t)=4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx$[1].

Becasue $\zeta(\frac{1}{2}+ti)=0$(t∈R and $t \neq 0$,s∈C) is ture, so $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$(t∈R and $t \neq 0$,s∈C)

and $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx=\xi(t)=0$,and

$\xi(t)=\frac{1}{2}-(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\ x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)=0$,so the roots of equations $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$ and $4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx=\xi(t)=0$ and $\xi(t)=\frac{1}{2}-(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\ x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)=0$ must all be real numbers.When $\zeta(s)$=0and $\xi(t)$=0, the real part of the equation $\xi(t)$=0 must be real between 0 and T.Because the real part of the equation $\xi(t)$=0 has the number of complex roots between 0 and T approximately equal to $\frac{T}{2\pi}\ln\frac{T}{2\pi}-\frac{T}{2\pi}$ [1] ,This result of Riemann's estimate of the number of zeros was rigorously proved by Mangoldt in 1895. Then,when $\zeta(s)$=0 and $\xi(t)$=0, the number of real roots of the real part of the equation $\xi(t)$=0 between 0 and T must be approximately equal to $\frac{T}{2\pi}\ln\frac{T}{2\pi}-\frac{T}{2\pi}$[1] ,so when the Riemann $\zeta(s)$function has nontrivial zeroes, then the Riemann hypothesis and the Riemann conjecture are perfectly valid.

Definition:

Assuming that a(n) is a uniproduct function, then the Dirichlet series $\sum_n a(n)n^{-s}$ is equal to the Euler product $\prod_p P(p, s)$ .Where the product is applied to all prime numbers p, it can be expressed as: $1+a(p)p^{-s}+a(p^2)p^{-2s}+\ldots$, this can be seen as a formal generating function, where the existence of a formal Euler product expansion and a(n) being a product function are mutually sufficient and necessary conditions. When a(n) is a completely integrative function, an important special case is obtained,where $P(p, s)$ is a geometric series, and $P(p, s)=\frac{1}{1-a(p)p^{-s}}$ .When $a(n)=1$,it is the Riemann zeta function, and more generally the Dirichlet feature.

Euler's product formula: for any complex number s,

$Rs(s) > 1$, then $\sum_n n^{-s} = \prod_p (1-p^{-s})^{-1}$,and when $Rs(s) > 1$ Riemann Zeta function $\zeta(s) = \sum_n n^{-s}=\prod_p (1-p^{-s})^{-1}(s \in C$ and $Rs(s) > 0 \wedge (s \neq 1)$,$n\in Z_+, p \in Z_+, s \in C$, n goes through all the natural numbers, p goes through all the prime numbers).

Riemann zeta function expression:

$\zeta(s)=1/1^s+1/2^s+1/3^s+\ldots+1/m^s$ (m tends to infinity, and m is always even).

(1)Multiply both sides of the expression by $(1/2^s)$,*for multiplication

$(1/2^s)*\zeta(s)=1/1^s*(1/2^s)+1/2^s*(1/2^s)+1/3^s*(1/2^s)+\ldots+1/m^s*(1/2^s)=1/2^s+1/4^s+1/6^s+\ldots+1/(2*m)^s$

This is given by (1) - (2)

$\zeta(s)-(1/2^s)*\zeta(s)=1/1^s+1/2^s+1/3^s+\ldots+1/m^s-[1/2^s+1/4^s+1/6^s+\ldots+1/(2*m)^s]$

The derivation of Euler product formula is as follows:

$\zeta(s)-(1/2^s)*\zeta(s)=1/1^s+1/3^s+1/5^s+\ldots+1/(m-1)^s$.

Generalized Euler product formula:

Suppose f(n) is a functionthat satisfies $f(n_1)f(n_2)=f(n_1n_2)$ and $\sum_n |f(n)| < +\infty$ (n1 and n2 are both natural numbers), then$\sum_n f(n)=\prod_p[1+f(p)+f(p^2)+f(p^3)+\ldots]$.

Proof:

The proof of Euler product formula is very simple, the only caution is to deal with infinite series and infinite products, can not arbitrarily use the properties of finite series and finite products. What I prove below is a more general result, and the Euler product formula will appear as a special case of this result.

Due to $\sum_n |f(n)| < \infty$, so $1+f(p)+f(p^2)+f(p^3)+\ldots$ absolute convergence.Consider the part of p<N in the continued product (finite product),Since the series is absolutely convergent and the product has only finite terms, the same associative and distributive laws can be used as ordinary finite summations and products.

Using the product property of f(n), we can obtain:

$\prod_{p<N}[1+f(p)+f(p^2)+f(p^3)+...]$=$\sum f(n)$.The right end of the summation is performed on all natural numbers with only prime factors below N (each such natural number occurs only once in the summation, because the prime factorization of the natural numbers is unique).Since all natural numbers that are themselves below N obviously contain only prime factors below N, So

Σ'f(n) = $\sum_{n<N} f(n)$ + R(N),Where R(N) is the result of summing all natural numbers that

are greater than or equal to N but contain only prime factors below N.From this we get:$\prod_{p<N}[1+f(p)+f(p^2)+f(p^3)+...]$=$\sum_{n<N} f(n)$ + R(N).For the generalized Euler product formula to hold, it is only necessary to prove $\lim_{n\to\infty} R(N)$=0,and this is obvious,because |R(N)| $\leq \sum_{n\geq N} |f(n)|$,and $\sum_n |f(n)| < +\infty$ sign of $\lim_{n\to\infty}\sum_{n\geq N} |F(n)|$ =0,thus $\lim_{n\to\infty} R(N)$=0.Beacuse $1+f(p)+f(p^2)+f(p^3)+...=1+$ $f(p)+f(p)^2+f(p)^3+...=[1-f(p)]^{-1}$, so the generalized Euler product formula can also be written as:

$\sum_n f(n)=\prod_p[1-f(p)]^{-1}$.In the generalized Euler product formula, take $f(n)=n^{-s}$,Then obviously $\sum_n |f(n)| < \infty$ corresponds to the condition Rs(s)>1 in the Euler product formula, and the generalized Euler product formula is reduced to the Euler product formula.

From the above proof, we can see that the key to the Euler product formula is the basic property that every natural number has a unique prime factorization, that is, the so-called fundamental theorem of arithmetic.

For any complex number s, $\chi(n)$ is the Dirichlet characteristic and satisfies the following properties:

1: There exists a positive integer q such that $\chi(n+q)=\chi(n)$;

2: when n and q are not mutual prime, $\chi(n)=0$;

3: $\chi(a)\chi(b)=\chi(ab)$ for any integer a and b;

If $0<Re(s)<1$,then

L(s, χ(n))=$\sum_1^\infty \frac{\chi(n)}{n^s}$ ($n\in Z_+, p\in Z_+, s\in C$, n goes through all the natural numbers, p goes through all the prime numbers, $\chi(n)\in R\wedge(\chi(n)\neq 0)$,$a(n)=a(p)=\chi(n)$),P(p, s)=$\frac{1}{1-a(p)p^{-s}}$).

Next we prove the generalized Riemann conjecture when the Dirichlet eigen function $\chi(n)$ is any real number that is not equal to zero,
and

$\eta(s)=\sum_1^\infty \frac{(-1)^{n-1}}{n^s}$ $(s\in C$ and $Rs(s)>0\wedge(s\neq 1))$ and $\eta(s)=(1-2^{1-s})\zeta(s)$$(s\in C$ and $Rs(s)>$ $0\wedge s\neq$ 1, $\zeta(s)$ is the Riemann Zeta function) ,so Riemann $\zeta(s)=\frac{\eta(s)}{(1-2^{1-s})}=\frac{1}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{n^s}=$ $\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-s})^{-1}$ $(s\in C$ and $Rs(s)>0\wedge(s\neq 1)$,$n\in Z_+, p\in Z_+, s\in C$, n goes through

all the natural numbers, p goes through all the prime numbers),
So

$GRH(s, ✗(n))=L(s, ✗(n))=\sum_n f(n) = \sum_1^\infty \frac{✗(n)}{n^s} = \sum_n a(n)n^{-s} = \prod_p P(p, s) = \prod_p(\frac{1}{1-a(p)p^{-s}})$

($n\in Z_+, p \in Z_+, s \in C$, n goes through all the natural numbers, p goes through all the prime numbers, $✗(n) \in R \wedge (✗(n) \neq 0), a(n) = a(p)= ✗(n)$),$P(p, s)=\frac{1}{1-a(p)p^{-s}}$).

$a(p)p^{-s}=a(p) * p^{-\rho} \frac{1}{(\cos(ylnp)+i\sin(ylnp))} =a(p) * (p^{-\rho}(\cos(ylnp) - i\sin(ylnp))$,

$(1 - a(p) * p^{-s})$ =1- $a(p) * (p^{-\rho}(\cos(ylnp) - i\sin(ylnp)) = 1 - a(p) * p^{-\rho} \cos(ylnp) + a(p) * ip^{-\rho}\sin(ylnp)$,

$a(p) * p^{-\bar{s}}=a(p) * p^{-\rho} \frac{1}{(\cos(ylnp)-i\sin(ylnp))} =a(p) * (p^{-\rho}(\cos(ylnp) + i\sin(ylnp))$,

$(1 - a(p)p^{-\bar{s}})$=1-$a(p)p^{-\rho} \cos(ylnp) - ia(p)p^{-\rho}\sin(ylnp)$,
because

$(1 - a(p) * p^{-s})=\overline{1 - a(p) * p^{-\bar{s}}}$,

so

$(1 - a(p)p^{-s})^{-1}=\overline{(1 - a(p)p^{-\bar{s}})^{-1}}$,

so

$\prod_p(1 - a(p)p^{-s})^{-1}=\overline{\prod_p(1 - a(p)p^{-\bar{s}})^{-1}}$,

becuse $L(s, ✗(n))=\sum_n a(n)n^{-s} = \prod_p(1 - a(p)p^{-s})^{-1}$ and $L(\bar{s}, ✗(n))=\sum_n a(n)n^{-\bar{s}} = \prod_p(1 - a(p)p^{-\bar{s}})^{-1}$, for the Generalized Riemann function

$L(s, ✗(n))=\sum_1^\infty \frac{✗(n)}{n^s} = \sum_n a(n)n^{-s} = \prod_p \frac{1}{1-a(p)p^{-s}}$ ($n\in Z_+, p \in Z_+, s \in C$, n goes through all the natural numbers, p goes through all the prime numbers, $✗(n) \in R \wedge (✗(n) \neq 0), a(n) = a(p)= ✗(n)$),$P(p, s)=\frac{1}{1-a(p)p^{-s}}$).

so

$L(s, ✗(n))=\overline{L(\bar{s}, ✗(n))}$.

$a(p)p^{1-s} = a(p)p^{(1-\rho-yi)} = a(p)p^{1-\rho}x^{-yi} = a(p)p^{1-\rho}(\cos(lnp) + i\sin(lnp))^{-y}=a(p)p^{1-\rho}(\cos(ylnp) - i\sin(ylnp))$,

$a(p)p^{1-\bar{s}} = a(p)p^{(1-\rho+yi)} = a(p)p^{1-\rho}p^{yi} = a(p)p^{1-\rho}(p^{yi}) = a(p)p^{1-\rho}(\cos(lnp) + i\sin(lnp))^{y}=a(p)p^{1-\rho}(\cos(ylnp) + i\sin(ylnp))$,
then

$a(p)p^{-(1-s)}=a(p)p^{\rho-1} \frac{1}{(\cos(ylnp)-i\sin(ylnp))} =a(p) * (p^{\rho-1}(\cos(ylnp) + i\sin(ylnp))$,

$(1-a(p)p^{-(1-s)})$=1-a(p)$p^{\rho-1}(\cos(ylnp)+isin(ylnp))$ =
1-a(p)$p^{\rho-1}\cos(ylnp)-a(p)p^{\rho-1}isin(ylnp)$,
$(1-a(p)p^{-\bar{s}})$ =1- $a(p)(p^{-\rho}(\cos(ylnp)+isin(ylnp))$ = $1-a(p)p^{-\rho}\cos(ylnp)-ia(p)p^{-\rho}sin(ylnp)$,

When $\rho=\frac{1}{2}$, then

$(1-a(p)p^{-(1-s)})$ =$(1-a(p)p^{-\bar{s}})$,
$(1-a(p)p^{-(1-s)})^{-1}$=$(1-a(p)p^{-\bar{s}})^{-1}$,
so
$\prod_p(1-a(p)p^{-(1-s)})^{-1}$=$\prod_p(1-a(p)p^{-\bar{s}})^{-1}$,

becuse $L(1-s,\chi(n))$=$\prod_p(1-a(p)p^{-(1-s)})^{-1}$ and $L(\bar{s},\chi(n))$=$\prod_p(1-a(p)p^{-\bar{s}})^{-1}$,

n∈ Z_+, p ∈ Z_+, s ∈C, n goes through all the natural numbers, p goes through all the prime

numbers, $\chi(n)$∈R∧ ($\chi(n)\neq 0$),$a(n)=a(p)=\chi(n)$),P(p, s)=$\frac{1}{1-a(p)p^{-s}}$.

so

Only $L(1-s,\chi(n))$=$L(\bar{s},\chi(n))$,

and

Only $L(1-\bar{s},\chi(n))$=$L(s,\chi(n))$.

Because $L(s,\chi(n))$=$\chi(n)\zeta(s)$ and $L(1-s,\chi(n))$=$\chi(n)\zeta(1\text{-}s)$, so When only $\rho=\frac{1}{2}$, it must be true that $L(s,\chi(n))$=$\overline{L(\bar{s},\chi(n))}$, and it must be true that $L(1-s,\chi(n))$=$L(\bar{s},\chi(n))$.

Suppose k∈ R,
$a(p)p^{k-s}$ = $a(p)p^{(k-\rho-yi)}$ = $a(p)*p^{k-\rho}x^{-yi}$ = $a(p)p^{k-\rho}(\cos(lnp)+i\sin(lnp))^{-y}$=$a(p)p^{k-\rho}(\cos(ylnp)-isin(ylnp))$,
$a(p)p^{k-\bar{s}}$=$a(p)p^{(k-\rho+yi)}$=$a(p)p^{k-\rho}p^{yi}$=$a(p)p^{1-\rho}(p^{yi})$ = $a(p)p^{k-\rho}(\cos(lnp)+i\sin(lnp))^{y}$=
$a(p)(p^{k-\rho}(\cos(ylnp)+isin(ylnp))$,
then

$a(p)p^{-(k-s)}$=$a(p)p^{\rho-k}\frac{1}{(\cos(ylnp)-isin(ylnp))}$ =$a(p)*(p^{\rho-k}(\cos(ylnp)+isin(ylnp))$,

$(1-a(p)p^{-(k-s)})$ =1- $(a(p)p^{\rho-k}*(\cos(ylnp)+isin(ylnp))$ = $1-a(p)*p^{\rho-k}\cos(ylnp)-ip^{\rho-k}sin(ylnp)$,
$(1-a(p)p^{-\bar{s}})$ =1- $(a(p)*p^{-\rho}(\cos(ylnp)+isin(ylnp))$ = $1-a(p)p^{-\rho}\cos(ylnp)-ia(p)p^{-\rho}sin(ylnp)$,

When $\rho=\frac{k}{2}$(k∈ R) , then

$(1-a(p)p^{-(k-s)})$ =$(1-a(p)p^{-\bar{s}})$,
$(1-a(p)p^{-(k-s)})^{-1}$=$(1-a(p)p^{-\bar{s}})^{-1}$,
so

$\prod_p(1-a(p)p^{-(k-s)})^{-1}=\prod_p(1-a(p)p^{-\bar{s}})^{-1}$,

becuse $L(k-s,\chi(n))=\prod_p(1-a(p)p^{-(k-s)})^{-1}$ and $L(\bar{s},\chi(n))=\prod_p(1-a(p)p^{-\bar{s}})$, for the

generalized Riemann function $L(s,\chi(n))(n\in Z_+, p\in Z_+, s\in C$, n goes through all the natural numbers, p goes through all the prime numbers,

$\chi(n)\in R\wedge(\chi(n)\neq 0), a(n)=a(p)=\chi(n)$),$P(p, s)=\frac{1}{1-a(p)p^{-s}}$).

so

Only $L(k-s,\chi(n))=L(\bar{s},\chi(n))$,

and

Only $L(k-\bar{s},\chi(n))=L(s,\chi(n))$.

And because Only $L(1-s,\chi(n))= L(\bar{s},\chi(n))$,so only k=1 be true.

so

$GRH(s,\chi(n))=L(s,\chi(n))=\sum_1^\infty\frac{\chi(n)}{x^s}=\frac{\chi(n)\eta(s)}{(1-2^{1-s})}=\frac{\chi(n)}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{x^s}=$
$\frac{\chi(n)}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{x^{\rho+yi}}=\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty\chi(n)(\frac{1}{x^\rho}*\frac{1}{x^{yi}})=$
$\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty\chi(n)(x^{-\rho})\frac{1}{(\cos(\ln x)+i\sin(\ln x))^y}=$
$\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty\chi(n)(x^{-\rho}(\cos(\ln x)+ i\sin(\ln x))^{-y})=\sum_1^\infty\chi(n)(x^{-\rho}(\cos(y\ln x)-$
$i\sin(y\ln x))$,

$$GRH(\bar{s},\chi(n))=L(\bar{s},\chi(n))=\sum_1^\infty\frac{\chi(n)}{n^{\bar{s}}}=\frac{\chi(n)\eta(\bar{s})}{(1-2^{1-s})}=\frac{\chi(n)}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{x^{\bar{s}}}$$

$$=\frac{\chi(n)}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{x^{\rho-yi}}=\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty\chi(n)(\frac{1}{x^\rho}*\frac{1}{x^{-yi}})$$

$$=\frac{1}{(1-2^{1-s})}\sum_1^\infty(\chi(n)\frac{1}{x^\rho}*\frac{1}{(\cos(\ln x)+i\sin(\ln x))^{-y}})$$

$$=\frac{1}{(1-2^{1-s})}\sum_1^\infty(\chi(n)*x^{-\rho}(\cos(\ln x)+ i\sin(\ln x))^y)=$$

$\frac{1}{(1-2^{1-s})}\sum_1^\infty(\chi(n)*x^{-\rho}(\cos(y\ln x)+i\sin(y\ln x))$,

$GRH(1-s,\chi(n))=L(1-s,\chi(n))=\sum_1^\infty\frac{\chi(n)}{x^{1-s}}=\frac{\chi(n)\eta(1-s)}{(1-2^{1-s})}=\frac{\chi(n)}{(1-2^{1-s})}\sum_1^\infty\frac{(-1)^{n-1}}{x^{1-\rho-yi}}=$

$\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty \chi(n)(\frac{1}{x^{1-\rho}}*\frac{1}{x^{-yi}}) = \frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_1^\infty(\chi(n)*x^{\rho-1}(\cos(y\ln x)+i\sin(y\ln x))$,

Suppose

$U=[\chi(n)1^{-\rho}Cos(yln1)-\chi(n)2^{-\rho}Cos(yln2)+\chi(n)3^{-\rho}Cos(yln3)-\chi(n)4^{-\rho}Cos(yln4)+...]$,

$V=[\chi(n)1^{-\rho}Sin(yln1)-\chi(n)2^{-\rho}sin(yln2)+\chi(n)3^{-\rho}sin(yln3)-\chi(n)4^{-\rho}sin(yln4)+...]$,

then

$L(s,\chi(n))=\overline{L(\bar{s},\chi(n))}$.

And x goes through all the natural numbers, so x=1,2,3... n-1,n ... ,let's just plug in, so

$L(s,\chi(n))=\sum_1^\infty\frac{\chi(n)}{n^s}=[\chi(n)1^{-\rho}Cos(yln1)-\chi(n)2^{-\rho}Cos(yln2)+\chi(n)3^{-\rho}Cos(yln3)-\chi(n)4^{-\rho}Cos(yln4)+...]-i[\chi(n)1^{-\rho}Sin(yln1)-\chi(n)2^{-\rho}sin(yln2)+\chi(n)3^{-\rho}sin(yln3)-\chi(n)4^{-\rho}sin(yln4)+...]=U-Vi$,

$U=[\chi(n)1^{-\rho}Cos(yln1)-\chi(n)2^{-\rho}Cos(yln2)+\chi(n)3^{-\rho}Cos(yln3)-\chi(n)4^{-\rho}Cos(yln4)+...]$,

$V=[\chi(n)1^{-\rho}Sin(yln1)-\chi(n)2^{-\rho}sin(yln2)+\chi(n)3^{-\rho}sin(yln3)-\chi(n)4^{-\rho}sin(yln4)+...]$,

Then

$L(\bar{s},\chi(n))=\sum_1^\infty\frac{\chi(n)}{n^{\bar{s}}}==[\chi(n)1^{-\rho}Cos(yln1)-\chi(n)2^{-\rho}Cos(yln2)+\chi(n)3^{-\rho}Cos(yln3)-4^{-\rho}Cos(yln4)+...]+i[\chi(n)1^{-\rho}Sin(yln1)-\chi(n)2^{-\rho}sin(yln2)+\chi(n)3^{-\rho}sin(yln3)-\chi(n)4^{-\rho}sin(yln4)+...]=U+Vi$,

$U=[\chi(n)1^{-\rho}Cos(yln1)-\chi(n)2^{-\rho}Cos(yln2)+\chi(n)3^{-\rho}Cos(yln3)-\chi(n)4^{-\rho}Cos(yln4)+...]$,

$V=[\chi(n)1^{-\rho}Sin(yln1)-\chi(n)2^{-\rho}sin(yln2)+\chi(n)3^{-\rho}sin(yln3)-\chi(n)4^{-\rho}sin(yln4)+...]$,

$L(s,\chi(n))$and $L(\bar{s},\chi(n))$ are complex conjugates of each other,that is $L(s,\chi(n))=\overline{L(\bar{s},\chi(n))}$.

When $\rho=\frac{1}{2}$,

then

$L(s,\chi(n))=L(1-s,\chi(n))=U-Vi$,

$U=[\chi(n)1^{-\rho}Cos(yln1)-\chi(n)2^{-\rho}Cos(yln2)+\chi(n)3^{-\rho}Cos(yln3)-\chi(n)4^{-\rho}Cos(yln4)+...]$,

$V=[\chi(n)1^{-\rho}Sin(yln1)-\chi(n)2^{-\rho}sin(yln2)+\chi(n)3^{-\rho}sin(yln3)-\chi(n)4^{-\rho}sin(yln4)+...]$.

And When $\rho=\frac{1}{2}$,

then

Only $L(1-s, \chi(n))$=$L(\overline{s}, \chi(n))$.

$GRH(k-s, \chi(n)) = L(k-s, \chi(n)) = \sum_1^{\infty} \frac{\chi(n)}{x^{k-s}} = \frac{\chi(n)\eta(k-s)}{(1-2^{1-s})} = \frac{\chi(n)}{(1-2^{1-s})} \sum_1^{\infty} \frac{(-1)^{n-1}}{x^{k-\rho-yi}} =$

$\frac{(-1)^{n-1}}{(1-2^{1-s})} \sum_1^{\infty} \chi(n)(\frac{1}{x^{k-\rho}} * \frac{1}{x^{-yi}}) = \frac{(-1)^{n-1}}{(1-2^{1-s})} \sum_1^{\infty}(\chi(n) * x^{\rho-k}(\cos(ylnx) + i\sin(ylnx))$,

W=[$\chi(n)1^{\rho-k}$Cos(yln1)− $\chi(n)2^{\rho-k}$Cos(yln2)+ $\chi(n)3^{\rho-k}$Cos(yln3)− $\chi(n)$ $4^{\rho-k}$Cos(yln4)+...]

U=[$\chi(n)1^{\rho-k}$Sin(yln1)− $\chi(n)2^{\rho-k}$sin(yln2)+ $\chi(n)3^{\rho-k}$sin(yln3)− $\chi(n)4^{\rho-k}$sin(yln4)+...] .

When $\rho=\frac{k}{2}$($k\in R$),

then

Only $L(k-s, \chi(n))$=$L(\overline{s}, \chi(n)) =$ W − Ui.

But the Riemann ζ(s)function only satisfies ζ(1-s)=$2^{1-s}\pi^{-s}$Cos($\frac{\pi s}{2}$)Γ(s)ζ(s),so when ζ(s)=0, then only $\zeta(1-s) = \zeta(s)$ =0, and when $\zeta(\overline{s})$ =0, then only $\zeta(1-s) = \zeta(\overline{s})$ =0, which is $\zeta(k-s)$=$\zeta(1-s) = \zeta(\overline{s})$,so only k=1 be true.so only Re(s)=$\frac{k}{2}$=$\frac{1}{2}$.

So Only $L(1-s, \chi(n)) = L(\overline{s}, \chi(n))$ is true, so only k=1 is true.

According the equation ζ(1-s)=$2^{1-s}\pi^{-s}$Cos($\frac{\pi s}{2}$)Γ(s)ζ(s) obtained by Riemann,since Riemann has shown that the Riemann ζ(s) function has zero, that is, in ζ(1-s)=$2^{1-s}\pi^{-s}$Cos($\frac{\pi s}{2}$)Γ(s)ζ(s), ζ(s)=0 is true. So only When ρ=$\frac{1}{2}$ and ζ(s)=0 and $\chi(n) \neq 0$, then $L(s, \chi(n))$= $\chi(n)$ζ(s)=0 is true.

Because $L(s, \chi(n))$= $\chi(n)$ζ(s) and $L(1-s, \chi(n))$= $\chi(n)$ζ(1-s), so When ρ=$\frac{1}{2}$, it must be true that $\overline{L(s, \chi(n))}$=$L(\overline{s}, \chi(n))$, and it must be true that $L(1-s, \chi(n))$=$L(\overline{s}, \chi(n))$.

According $\zeta(1-s)$ =ζ(s)=0 and ζ(s)=ζ($\overline{s}$)=ζ(1-$\overline{s}$)=0, so $L(s, \chi(n)) = L(1-s, \chi(n))$ =0 and $L(s, \chi(n))$=$L(\overline{s}, \chi(n))$=$L(1-\overline{s}, \chi(n))$=0,then s=$\overline{s}$ or s=1-s or $\overline{s}$=1-s ,so s∈R, or ρ+yi=1-ρ-yi ,or ρ-yi=1-ρ-yi, so s ∈ R, or ρ=$\frac{1}{2}$and y=0,or $\rho = \frac{1}{2}$ and y ∈R and y≠ 0, so s∈ R for example s=-2n(n∈ Z_+), or s=$\frac{1}{2}$+oi ,or s=$\frac{1}{2}$+yi(y ∈R and y ≠ 0).ζ($\frac{1}{2}$) > □(1) = ∞,drop it, when s=-2n(n∈ Z_+),it's the trivial zero of the Riemann ζ(s) function, drop it.So only $s = \frac{1}{2} + yi$ (y ∈ R , and y ≠ 0, s ∈

C) is true, or say $s=\frac{1}{2}+ti$ ($t \in R$, and $t \neq 0, s \in C$) is true.And beacause only when $\rho=\frac{1}{2}$,the next three equations, $L(\rho+yi, \chi(n))=0$, $L(1-\rho-yi, \chi(n))=0$ and $L(\rho-yi, \chi(n))=0$ are all true. And because $L(\frac{1}{2}, \chi(n))=\infty$, so only $s=\frac{1}{2}+yi$ ($y \in R$ and $y \neq 0, s \in C$) is true,or say only $s=\frac{1}{2}+ti$ ($t \in R$ and $t \neq 0, s \in C$) is true.

The Generalized Riemann hypothesis and the Generalized Riemann conjecture must satisfy the properties of the $L(s, \chi(n))$ function, The properties of the $L(s, \chi(n))$ function are fundamental, the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct to reflect the properties of the $L(s, \chi(n))$ function , that is, the roots of the $L(s, \chi(n))=0$ can only be $s=\frac{1}{2}+ti$($t \in C, s \in C$ and $t \neq 0$), that is, Re(s) must only be equal to $\frac{1}{2}$, and Im(s) must be real, and Im(s) is not equal to zero.So the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct.

According $L(1-s, \chi(n))= L(s, \chi(n))=0$,so the zeros of the$L(s, \chi(n))$function in the complex plane also correspond to the symmetric distribution of point ($\frac{1}{2}$,0i) on a line perpendicular to the real number line in the complex plane,so When $L(1-s, \chi(n)) = L(s, \chi(n)) = 0$,s and 1-s are pair of zeros of the function $L(s, \chi(n))$ symmetrically distributed in the complex plane with respect to point ($\frac{1}{2}$,0i) on a line perpendicular to the real number line of the complex plane.

We got $\overline{L(s, \chi(n))} = L(\overline{s}, \chi(n))$($s=\rho+yi, \rho \in R$, $y \in R$ and $y \neq 0$) before,When t in Generalized Riemann's hypothesis $s=\frac{1}{2}+ti$($t \in C, s \in C$ and $t \neq 0$) is a complex number, and $s=\frac{1}{2}+ti=\rho+yi$, then s in $\overline{L(s, \chi(n))} = L(\overline{s}, \chi(n))$ ($s=\rho+yi, \rho \in R$, $y \in R$ and $y \neq 0$) is consistent with s in Generalized Riemann's hypothesis $s=\frac{1}{2}+ti$($t \in C, s \in C$ and$t \neq 0$).when $\overline{L(s, \chi(n))} = L(\overline{s}, \chi(n))=0$($s=\rho+yi, \rho \in R$, $y \in R$ and $y \neq 0$),since s and $\overline{s}$ are a pair of conjugate complex numbers, so s and $\overline{s}$ must be a pair of zeros of the Generalized function $L(s, \chi(n))$in the complex plane with respect to point (ρ,0i) on a line perpendicular to the real number line.s is a symmetric zero of 1-s, and a symmetric zero of $\overline{s}$. By the definition of complex numbers, how can a symmetric zero of the same Generalized Riemann function $L(s, \chi(n))$ of the same zero independent variable s on a line perpendicular to the real number axis of the complex plane be both a symmetric zero of 1-s on a line perpendicular to the real number axis of the complex plane with respect to point ($\frac{1}{2}$,0i) and a symmetric zero of $\bar{s}$ on a line perpendicular to the real number axis of the complex plane with respect to point (ρ,0i)?Unless ρ and $\frac{1}{2}$ are the same value, is also that $\rho = \frac{1}{2}$, and only 1-s=$\bar{s}$ is true, only s=$\frac{1}{2}$+ti (t∈R and t ≠ 0,s∈C) is true. Otherwise it's impossible,this is determined by the uniqueness of the zero of Generalized Riemann function L(s, ✗(n)) on the line passing through that point perpendicular to the real number axis of the complex plane with respect to the vertical foot symmetric distribution of the zero of the line and the real number axis of the complex plane,Only one line can be drawn perpendicular from the zero independent variable s of Generalized Riemann function L(s, ✗(n))on the real number line of the complex plane, the vertical line has only one point of intersection with the real number axis of the complex plane. In the same complex plane, the same zero point of Generalized Riemann function L(s, ✗(n)) on the line passing through that point perpendicular to the real number line of the complex plane there will be only one zero point about the vertical foot symmetric distribution of the line and the real number line of the complex plane,so I have proved the generalized Riemann conjecture when the Dirichlet eigen function ✗(n) is any real number that is not equal to zero,Since the nontrivial zeros of the Riemannian function ζ(s) and the generalized Riemannian function L(s, ✗(n)) are both on the critical line perpendicular to the real number line of Re(s)=$\frac{1}{2}$ and Im(s)≠ 0, these nontrivial zeros are general complex numbers of Re(s)=$\frac{1}{2}$ and Im(s)≠ 0,so I have proved the generalized Riemann conjecture when the Dirichlet eigen function ✗(n) is any real number that is not equal to zero.

The Generalized Riemann hypothesis and the Generalized Riemann conjecture must satisfy the properties of the L(s, ✗(n)) function, The properties of the L(s, ✗(n)) function are fundamental, the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct to reflect the properties of the L(s, ✗(n))function, that is, the roots of the L(s, ✗(n))=0 can only be s=$\frac{1}{2}$+ti(t∈C,s∈C and t ≠ 0), that is, Re(s) can only be equal to $\frac{1}{2}$, and Im(s) must be real, and Im(s) is not equal to zero.So the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct.

The Generalized Riemann hypothesis and the Riemann conjecture are perfectly valid, and the Polygnac conjecture and the twin prime conjecture and Goldbach's conjecture must satisfy the properties of the Generalized Riemann ζ(s) function and the Riemann ξ(s) function.

When $L(s, \chi(n)) = 0$ ($n \in Z_+, p \in Z_+, s \in C$, n goes through all the natural numbers, p goes through all the prime numbers ,

$\chi(n) \in R \wedge \chi(n) \neq 0$), $a(n) = a(p) = \chi(n)$), $P(p, s) = \frac{1}{1-a(p)p^{-s}}$),and $s = \frac{1}{2} + ti$ ($t \in R$ and $t \neq 0, s \in C$), for any complex number s,when $\chi(n)$ is the Dirichlet characteristic and satisfies the following properties:

1: There exists a positive integer q such that $\chi(n+q) = \chi(n)$ ($n \in Z_+$);

2: when n and q are not mutual prime, $\chi(n)=0$ ($n \in Z_+$);

3: $\chi(a)\chi(b)=\chi(ab)$ ($a \in Z_+, b \in Z_+$)for any integer a and b;

Suppose q=2k($k \in Z_+$), if n and n+q are all prime number,

then $\chi(n+q) = \chi(n) = \chi(p) \equiv 1$($n, n+q$, and p , they all go through all the prime numbersr),

when n and q are not mutual prime, $\chi(n)=0$($n \in Z_+$),and $\chi(a)\chi(b)=\chi(ab)$ ($a \in Z_+, b \in Z_+$, a and b are all prime number) for any prime number a and b,then the three properties described by the Dirichlet eigenfunction $\chi(n)$ above fit the definition of the Polignac conjecture, the Polignac conjecture states that for all natural numbers k, there are infinitely many pairs of prime numbers (p,p+2k)($k \in Z_+$). In 1849, the French mathematician A. Polignac proposed the conjecture.When k=1, the Polygnac conjecture is equivalent to the twin prime conjecture.In other words, when $L(s, \chi(n)) = 0$($n \in Z_+, p \in Z_+, s \in C$, n goes through all the natural numbers, p goes through all the prime numbers, $\chi(n) \in R \wedge (\chi(n) \neq 0)$, $a(n) = a(p) = \chi(n)$), $P(p, s) = \frac{1}{1-a(p)p^{-s}}$), and generalized Riemann hypothesis and the generalized Riemann conjecture are true, then the Polygnac conjecture must be completely true, and if the Polignac conjecture must be true, then the twin prime conjecture and Goldbach's conjecture must be true.I proved that the generalized Riemannian hypothesis and the generalized Riemannian conjecture are true, so when $L(s, \chi(n)) = 0$($n \in Z_+, p \in Z_+, s \in C$, n goes through all the natural numbers, p goes through all the prime numbers,

$\chi(n)) \in R \wedge (\chi(n)) \neq 0)$, $a(n) = a(p) = \chi(n)$), $P(p, s) = \frac{1}{1-a(p)p^{-s}}$) and $s = \frac{1}{2} + ti$ ($t \in R$ and $t \neq 0, s \in C$) ,I also proved that the Polignac conjecture,twin prime conjecture must be true and Goldbach conjecture are completely or almost true.The Generalized Riemann hypothesis and the Riemann conjecture are perfectly valid, so the Polygnac conjecture and the twin prime conjecture and Goldbach's conjecture must satisfy the properties of the Generalized Riemann ζ(s) function and the Riemann ξ(s) function, so the Polignac conjecture,twin prime conjecture must be true and Goldbach conjecture is almost or completely true.

Riemann hypothesis and the Riemann conjecture are completely correct and the Generalized Riemann hypothesis and the Generalized Riemann conjecture are completely correct and the Polignac conjecture,twin prime conjecture must be tue and Goldbach conjecture are almost or completely true.

III. Conclusion

After the Riemann hypothesis and the Riemann conjecture and the Generalized Riemann hypothesis and the Generalized Riemann conjecture are proved to be completely valid, the research on the distribution of prime numbers and other studies related to the Riemann hypothesis and the Riemann conjecture will play a driving role. Readers can do a lot in this respect.

IV.Thanks

Thank you for reading this paper.

V.Contribution

The sole author, poses the research question, demonstrates and proves the question.

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